PA -MDC Grade 5 Math: time for recess

This lesson is intended to measure student understanding of multiplication of fractions by solving a real-world problem that will show understanding of interpreting a fraction as division of the numerator by the denominator and multiplying fractions using an area model and applying the standard algorithm

This lesson is modeled from unit taken from EngageNY and based on lesson created by John A. Van de Walle

PROBLEM SOLVING FORMATIVE ASSESSMENT LESSON

Grade 5 Math: TIME FOR RECESS

**Mathematical Goals**

This lesson is intended to measure student understanding of multiplication of fractions by:

* Solving a real-world problem that will show understanding of interpreting a fraction as division of the numerator by the denominator.
* Multiplying fractions using an area model and applying the standard algorithm.

**Common Core Standards**

**5. NF.4a** —Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Interpret the product (*a*/*b*) × *q* as a part of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*.

**5. NF.4b** —Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

**PA- Core**

**PA CC2.1.5**.**C.2 -** Apply and extend previous understandings of multiplication and division to multiplying and dividing fractions.

**Standards for Mathematical Practice:** This lesson also relates to **all** the Standards for Mathematical Practice with a particular emphasis on Practices 1, 3, and 6.

**MP.1 –** Make sense of problems and persevere in solving them

**MP.3 –** Construct viable arguments and critique the reasoning of others

**MP.6 –** Attend to precision

**Introduction**

This lesson is structured in the following way:

* A few days before the FAL, students tackle the problem individually. You review their work and write questions to help students improve their solutions.
* At the beginning of the first day, students respond to your questions. Then they work collaboratively in pairs to produce a better solution. In this lesson it is best to work in pairs and then have teams of 4 combine to discuss solutions and again choose the best and final solution to the problem.
* Groups present their solutions and evaluate and comment on sample solutions, followed by a whole-class discussion about the work
* Finally, a follow-up to the lesson where students reflect, review and evaluate their work on the problem.

**Materials**

* Each student will need a white-board, a pen, an eraser, a copy of the pre-assessment sheet, *Time for Recess and Some lunch*
* Each small group of students will need a large sheet of paper for making a poster
* Provide rulers, pencils, graph and plain paper, geo-board/rubber band, sq. color tiles to manipulate and display (promote their use in convincing each other of the correct response.)

**Time Needed**

Timings are approximate: exact timings will depend on the needs of your students

Suggested: Lesson estimated 90 – 120 minutes

* No more than 15 minutes for the pre-assessment
* 10 minute review of questions posed individually
* 20 minute discussion with partner, 30 minute discussion with group of 4 to summarize and complete chart
* 15 minute whole class discussion 2 or 3 students summarizing results
* Collaborative analysis and share out 20 minutes
* Reflection 10 minutes or homework

**Before the Lesson**

**Pre-Assessment: *Time for Recess (no more than 15 minutes)***

Have students work on the assessment task a few days before the formative assessment lessons. This will give you an opportunity to assess the work, find out about students’ different problem-solving approaches, and notice any difficulties students experience with the task. You should then be able to target your help more effectively in the lessons that follow.

Give each student a copy of the pre-assessment *Time for Recess and Some Lunch*

*Introduce the task and help students to understand what you are asking them to do.*

*Today you are going to solve a playground dilemma of what fraction of the playground is set aside for soccer?*

*There are many ways to tackle the problem and there is more than one way to express and prove your answer. You are to decide on the math to use.*

*Read through the task carefully and answer the questions. Make sure you record all your reasoning really carefully – explain all your decisions.*

*I have more paper and other materials you might wish to use.*

*If available post a picture of a playground that has different areas of play for students to help them see what you are asking them to do.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. Some students may find it difficult to get started: be aware that if you offer help too quickly, students will merely do what you say and will not think for themselves. If, after several minutes, students are still struggling, try to help them understand what is required. The first few questions on the Common issues table may be useful.

Students who sit together often produce similar answers so that when they come to compare their work, they often have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions because students then have varied approaches to discuss.

When all students have made a reasonable attempt at the task, reassure them that they will have time to revisit and revise their solutions later.

**Assessing students’ responses**

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. We suggest that you do not score students’ work. Research shows that this will be counter-productive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the Common issues table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students’ work. We recommend you either:

• write one or two questions on each student’s work, or

• give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the start of the first lesson.

**Common issues Suggested questions and prompts**

|  |  |
| --- | --- |
| **Has difficulty getting started**  **For example: The student writes or draws very little**  **Or the student draws a shape not relevant to problem or dimensions given.**  **Or student has difficulty because he states there is not enough information to solve the problem** | * **What are you trying to do? Explain the task in your own words.** * **What math have you worked on before that might help you?** * **What do you know? What information do you need?** * **What are you trying to find out –what is being asked in this problem?** |
| **The student demonstrates confusion of fractional concepts**  **-he attempts to tile the rectangle ,tiles are not uniform**  **– he is disorganized in his approach ( precision lacking)** | * **Are your tiles organized and similar in size and shape?** * **Are there any of the tools available that would help you be more organized in your approach?** * **What did you do first?Why?** |
| **The students answer is inaccurate**  **He uses the wrong fractions and does not complete the problem( 2/3 of 1/5)**   * **No model** * **No evidence of understanding the task** | * **Can you explain why you used the fractions that you chose?** * **Does your answer seem reasonable? Explain** * **Can you draw a picture of your number sentence?** |
| **Does not justify his answer**  **Only area model with no fractional explanation or number sentence** | * **Can you explain through words and number sentences how you determined the area model?** * **How do you know that is the correct model?** |
| **The student finds an incorrect area of a rectangle with fractional sides**   * **student broke down the shape into appropriate tiles** * **Did not recognize “of the remaining”** | * **Count the tiles in your array; do they match your fractions’ numerator and denominator?** * **Reread the word problem; are there key words that you may have skipped over that may impact your answer?** * **How does your picture represent your written response ?** |
| **Provides a complete, well-justified response** | * **Can you find the area of a rectangle with a length of 3/7 in and a width of 2/3 in. using both a model and a number sentence? Explain where your number sentence is represented in your model.** |

Please use the blanks to add your class misconceptions and directed questions.

When a student has completed the pre-assessment with ease you can create a challenge as above. It is suggested that you create a card to attach to their pre-assessment and after they complete it to share with other student or to wait for whole class discussion and present.

**Can you find the area of a rectangle with a length of 3/7 in and a width of 2/3 in. using both a model and a number sentence? Explain where your number sentence is represented in your model.**

**This can also be used when a team completes the assignment earlier than their peers.**

**Suggested lesson Outline**

**Reviewing individual solutions to the task (10 minutes)**

**Teacher: Recall the *Time for Recess* problem we were working on previously?**

**Today you are going to work together to try to improve your initial attempts at this task. First, I have had a look at your work and have some questions I would like you to think about. On your own, carefully read through the questions I have written. I would like you to use the questions to help you to think about ways of improving your own work. Use your mini-whiteboards to make a note of anything you think will help to improve your work.**

**Return your students’ work on *Time for Recess.***

If you have not added questions to individual pieces of work or highlighted questions on a printed list of questions then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

**Collaborative Activity producing a joint solution (35 – 40 minutes)**

Organize students into groups of two and or a single group three. Grouping students who have taken different approaches but have similar level of understanding may lead to more profitable discussions. Allow 10 minutes for pair discussion and 25 minutes for group of 4 to choose method and write on chart paper

**Planning a Joint Method**

Create a slide, laminate handout for tables or keep posted where students can follow the steps of planning a joint solution:

**Planning a Joint Solution**

1. **Take turns to explain your original method and how you think your work could be improved having considered my feedback.**
2. **Listen carefully to each other and ask questions if you don’t understand.**
3. **Once you understand each other’s work agree together in your group on the best approach for completing the problem.**
4. **Make sure that everyone in the group can explain the reason for your chosen method.**
5. **Outline on your large sheet of paper the approach you are going to use.**

To confirm students know what they have to do, ask a couple of students to explain, in succession, the different steps of the activity. Once students have had chance to agree together on their joint method, it may be helpful to ask a few groups to share their planned method for their joint solution. It is important that students think carefully about which method to use and do not simply revert to one of the individual methods. The aim is to produce a joint solution that is better than either of the individual responses.

* What method do you plan to use? In what ways is this approach different to your individual methods?
* How has looking at other strategies in your group influenced your thinking?
* Each member of the group should be able to explain the reasons for their chosen method.
* The group should now to turn their large sheet of paper over /write below representation and write their joint solution clearly on the poster paper.

While students work in small groups you have two tasks: to note their different approaches to the task and to support student problem solving.

**Support student problem solving**

If students are struggling to produce a joint solution to the task, try not to make suggestions that move them towards a particular approach. Instead, ask questions that help them to clarify their thinking, focusing on the strategies rather than the solution. Encourage students to justify any decisions they make.

* What have you found out so far?
* What decisions do you need to make?
* How can you systematically consider the possibilities?
* What is your strategy? What do you need to do next?
* What can you do to better organize your thinking?

You may want to use the questions in the Common Issues table to support your questioning. If the whole class is struggling on the same issue, you could write one or two relevant questions on the board or hold a brief whole-class discussion.

**Sharing posters (15 minutes)**

When students have had sufficient time to work on their posters, give them the opportunity to compare their work, by one person from each group visiting the poster of another group. The visiting student should try to make sense of what is on the poster and the assumptions the group has made. The group being visited should explain their work to the visitor. Next choose 2 or 3 groups to present their work. Include the challenge question for all students to see and hear.

**Collaborative analysis of *Sample Responses to discuss (20 - 25 minutes)***

Distribute copies of *Sample Responses to Discuss* to each group of students. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

**There may not be time, and it is not essential, for all groups to look at all four sample responses.** If this is the case, be selective about what you hand out. For example, groups that have successfully completed the task using one method will benefit from looking at a different approach. Other groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

Teacher says: In your groups you are now going to look at some student work on the task. Notice in what ways this work is similar to yours and in which ways it is different. There are some questions for you to answer as you look at the work. You may want to add notes to the work to make it easier to follow.

Encourage students to focus on evaluating the strategies and math in the student work, not on superficial features, such as whether or not the student has neat handwriting. During the small group work, support the students as in the first collaborative activity. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the collaborative group work.

**Whole-class discussion: comparing different approaches (15 minutes)**

Hold a whole-class discussion to consider the different approaches used in the sample work. Focus the discussion on parts of the task students found difficult. Ask the students to compare the different solution methods.

* Which approach did you think was the fairest and most effective? Why?
* Which approach did you find most difficult to understand? Why?
* How could the student improve his/her answer?
* Did anyone come up with a method different from these?

Try to focus the discussion on any common misconceptions you noticed in the collaborative work. You may want to draw on the questions in the Common issues table to support your own questioning. Try to resist simply explaining the mistakes students have made.

**Follow-up lesson: review solutions to *Time for Recess* (10 minutes)**

Give out the *sheet How Did You Work?* ask students to complete this questionnaire. The questionnaire should help students review their progress. If you have time you may also want to ask your students to read through their original solutions and using what they have learned, attempt the task again, perhaps using a different method. In this case, give each student a blank copy of the assessment *Time for Recess.*

**Some teachers give this task for homework.**

TIME FOR RECESS

The 4th and 5th graders both have outdoor recess at the same time. The

4th graders like to play basketball, the 5th graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Create an area model below to show what fraction of the entire playground is set aside for soccer. Defend your diagram using a mathematical equation and written explanation.



TIME FOR RECESS ANSWER KEY

1. The 4th and 5th graders both have outdoor recess at the same time. The

4th graders like to play basketball, the 5th graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play



soccer. Use the model below to show what fraction of the entire

playground is set aside for soccer. Defend your diagram using a mathematical equation and written explanation.

PLAYGROUND ANSWER: The diagram at the left was tiled appropriately. The shaded

region is clearly 8/15.

The correct number sentence is:

4/5 X 2/3 = 8/15

2/3 of the yard.

The written explanation should include how the student tiled the model, why they shaded the region they shaded, and how it relates to the

number sentence.

4/5 of the yard remaining after 1/5 is

used for students to read.

8/15 of the yard is the area remaining for students to play soccer.

**Challenge Question/Solution**

Please find the area of rectangle with a length of 3/7 inch and a width of 2/3 inch below using both a model and a number sentence. Explain where your

number sentence is represented in your model.

Number Sentence:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  | | |  |  |  |  |
|  |  |  |  |

A = L X W

A = 3/7 in. X 2/3 in. A = 6/21 sq. in.

A = 2/7 sq. in.

2/3 in.

3/7 in.

Answer: The area of a rectangle with a length of 3/7 inch and a width of 2/3 inch is 6/21, or 2/7 square inches

when simplified. I proved this by tiling a model and using the area formula. Both my diagram and number

sentence match. I’ve indicated the two numbers I was multiplying as my fractional sides in my diagram. The

area 2/7 square inches is the shaded portion of my diagram.

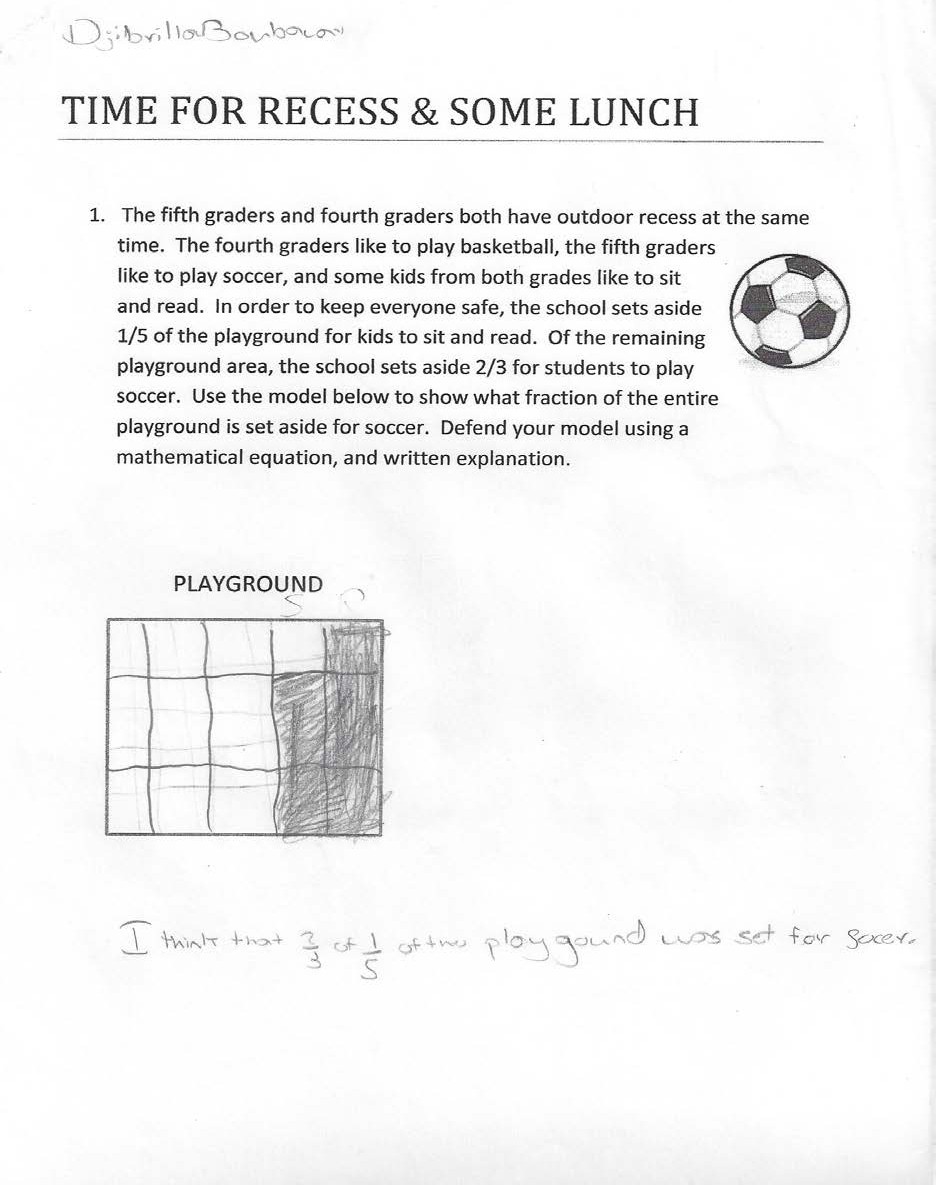
GRADE 5 MATH:

TIME FOR RECESS

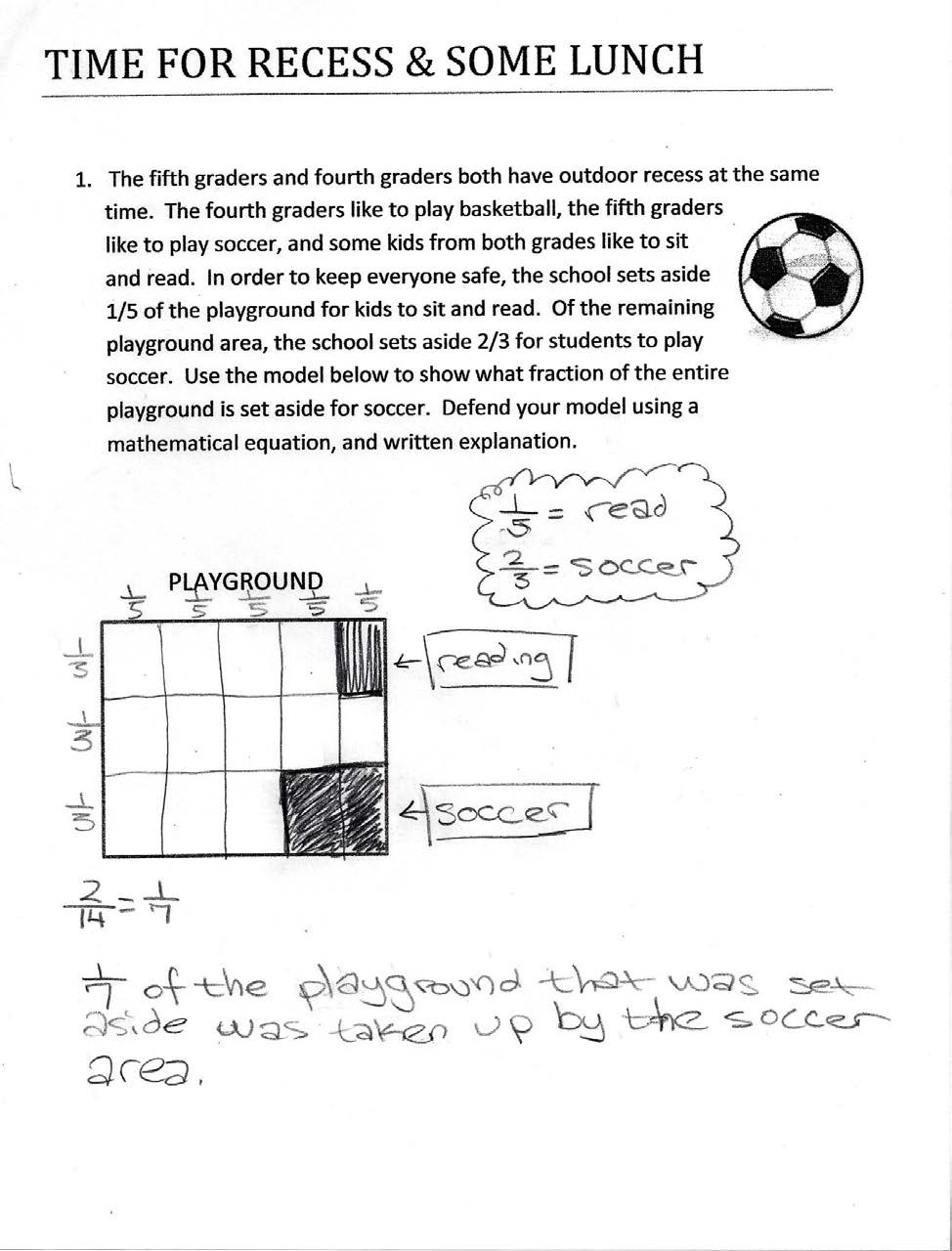
ANNOTATED STUDENT WORK

This section contains annotated student work. The student work shows examples of student understandings and misunderstandings of the task. Your students’ will do an error analysis and make suggestions for misconceptions or errors they find.

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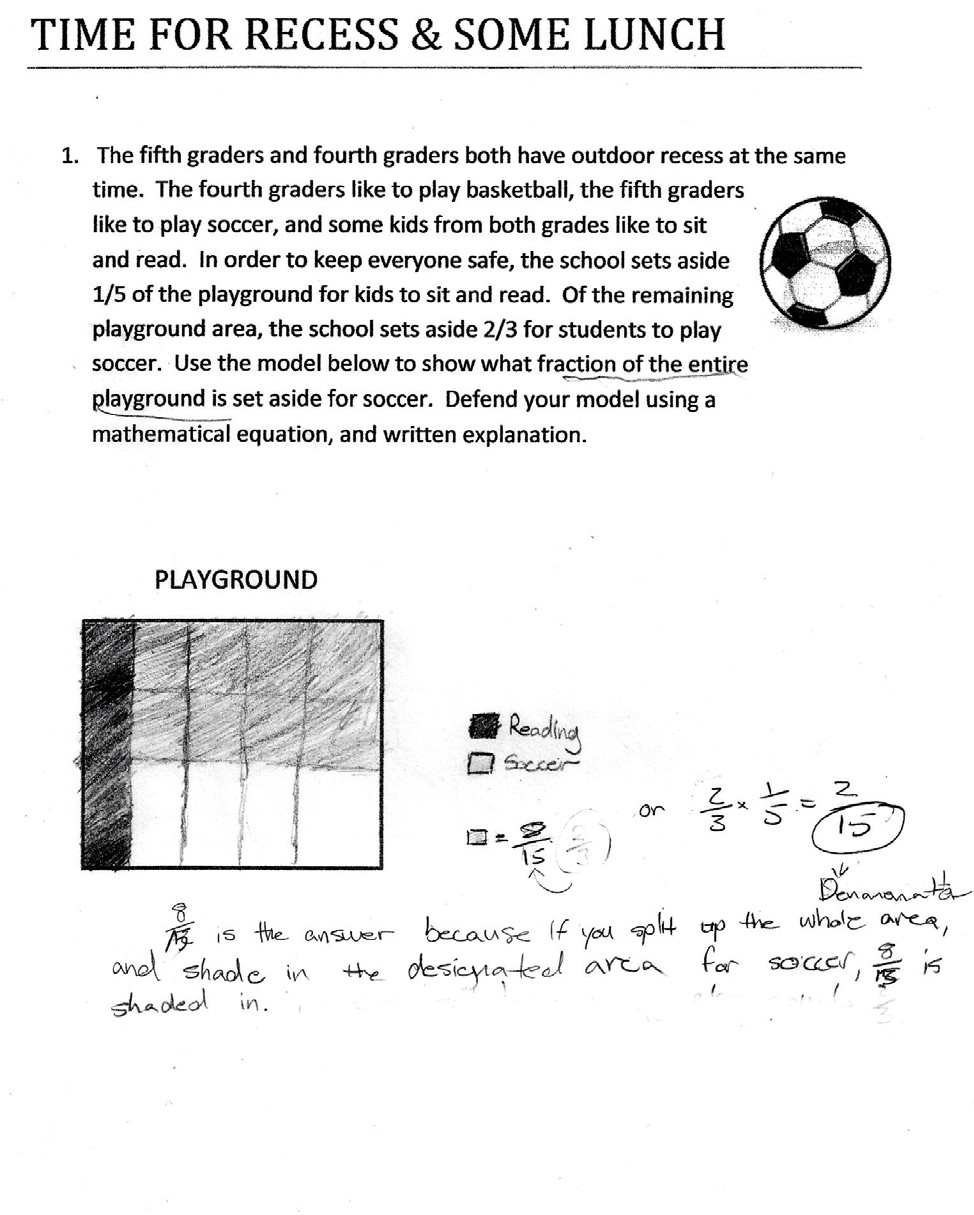
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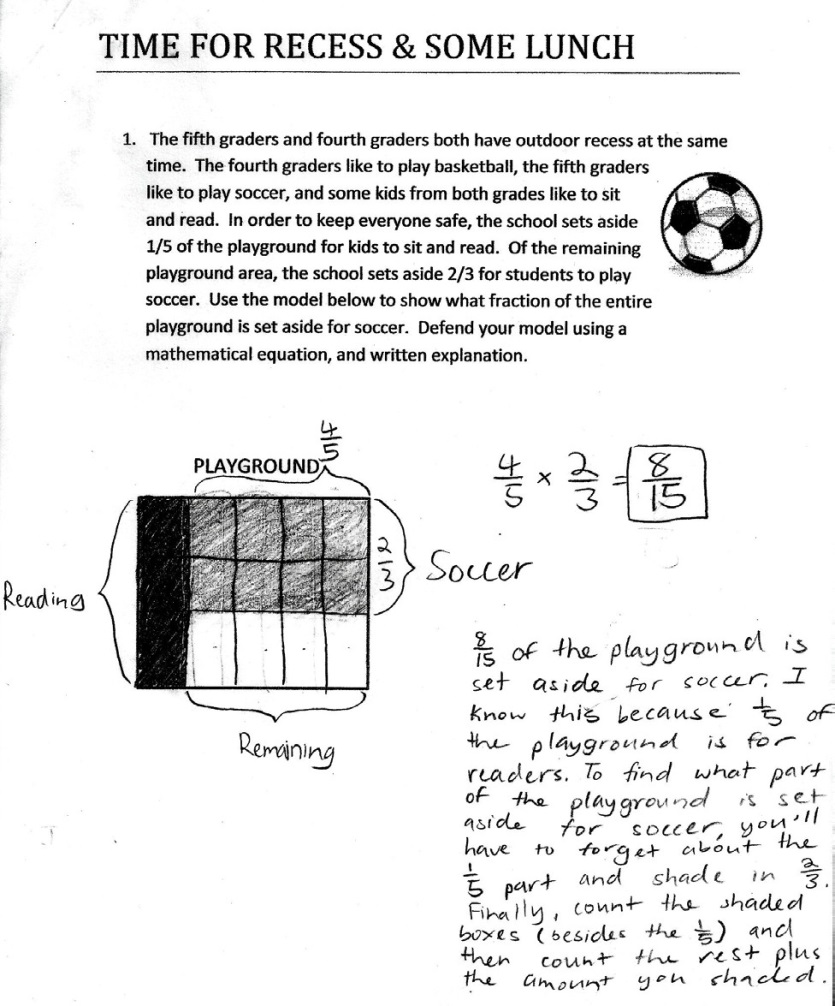
STUDENT RESPONSE 1



**Student Response 2**

**Student Response 3**





**Student Response 4**

**Teacher Notes**

Student 1: Annotated Student Work Summary

* The student demonstrates only minor fractional understanding.
* They do not recognize fractions as division of the numerator by the denominator (5.NF.3).
* They do not demonstrate major understanding of the concept of finding a fraction of a fraction by multiplying or using a fraction model (5.NF.4a).
* They do not demonstrate major understanding of the concept of area of a rectangle by multiplying fractional side lengths or tiling (5.NF.4b).

The student needs to revisit these concepts, both to improve the use of modeling, as well as to improve the understanding of the basic algorithm for multiplying fractions.

Student 2: Annotated Student Work Summary

* The student demonstrates some fractional understanding.
* They recognize fractions as division of the numerator by the denominator, but have some inaccuracies when tackling problems involving this understanding (5.NF.3).
* They can create fractional models that are tiled correctly, but are missing the connections needed to shade the appropriate tiles (5.NF.4a).
* They demonstrate some understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths or tiling (5.NF.4b).
* They tile correctly, but once again are not making the connections needed to shade the appropriate amount. They also use the wrong operation (addition) when applying the area formula.

The student needs to revisit these concepts, both to improve the application of models, and to build a better understanding of the area formula.

Student 3: Annotated Student Work Summary

* The student demonstrates sound fractional understanding.
* They recognize fractions as division of the numerator by the denominator, but have some minor inaccuracies when tackling problems involving this understanding (5.NF.3).
* They can create fractional models that are tiled correctly, and apply the correct algorithm to arrive at the correct answer when multiplying fractions, but have minor inaccuracies in their work, or their explanation is incomplete (5.NF.4a).
* They demonstrate sound understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths and tiling (5.NF.4b).
* They tile correctly, and apply the area formula, but their written explanation is the missing component that would tie the two concepts together.

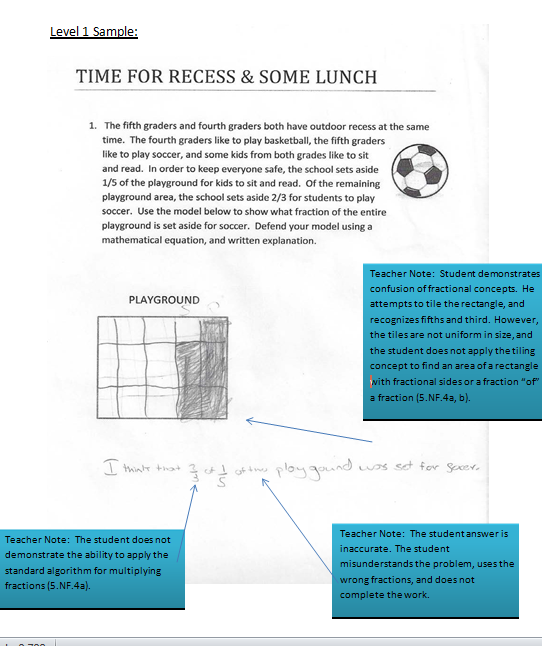
The student needs to work on fluency and accuracy in their details, as well as improving their written explanations.

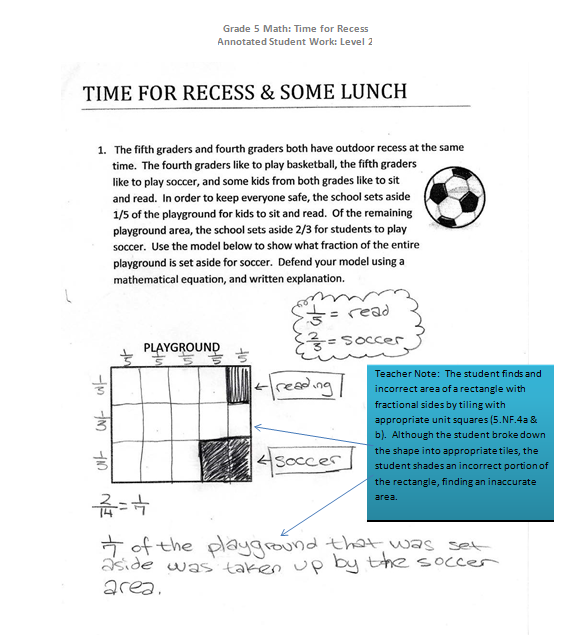
Student 4: Annotated Student Work Summary

* The student demonstrates consistent deep fractional understanding.
* They recognize fractions as division of the numerator by the denominator, and apply this understanding to solve word problems involving whole numbers that have fractional products (5.NF.3).
* They can create fractional models that are tiled correctly, and apply the correct algorithm to arrive at the correct answer when multiplying fractions accurately, with complete, clear explanations (5.NF.4a).
* They demonstrate consistent deep understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths and tiling (5.NF.4b).
* They tile correctly, apply the area formula, and include a clear, detailed written explanation that ties the two concepts together.

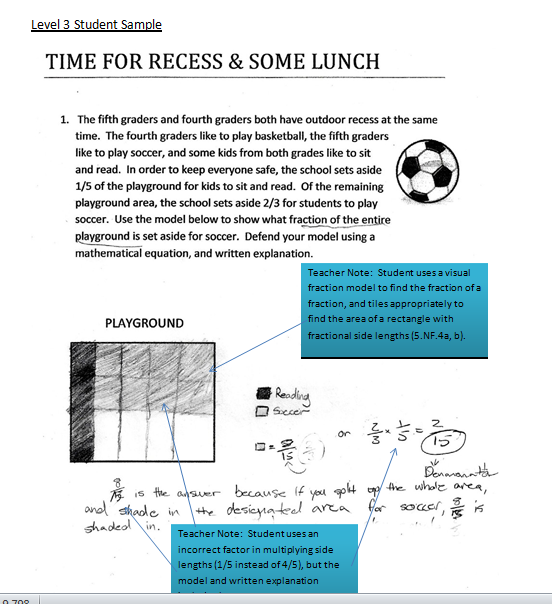
The student consistently exhibits a deep understanding of all content standards in this task.

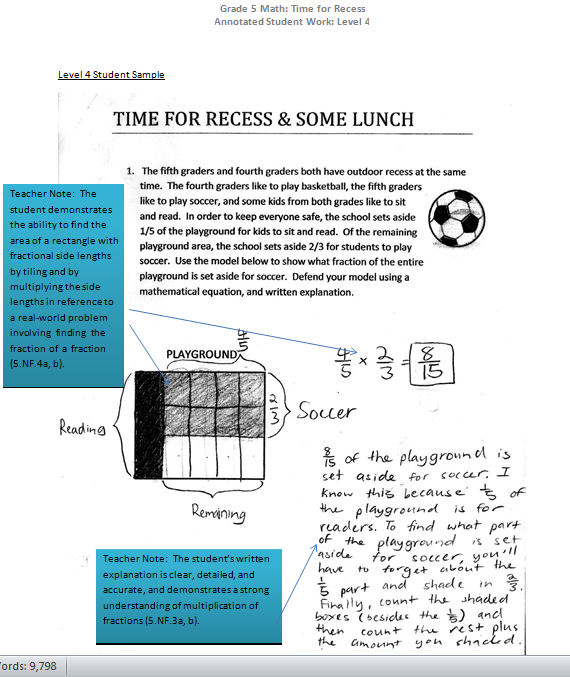
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**Level 2 sample**





**Cheat-Sheet with Suggestions for Helping Students Access Information**

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| **Barrier to Learning** | **Suggested Strategy** |
| **Student lacks understanding of math language** | Review domain specific vocabulary; create a picture dictionary with student. |
| **Student lacks basic fractional knowledge** | Review prior lessons in fractions; use manipulatives to explore fraction concepts. |
| **Students have problems in understanding math word problems (reading comprehension)** | Use key words and "picture stories" to help students identify the appropriate operation. |
| Build vocabulary through repeated classroom use and picture dictionary. |
| Work on reading and understanding problems through modeling in small groups and peer-to- peer situations. |
| **Student struggles with multi-step problems** | Break the problem into smaller tasks, an understandable sequence. |
| **Student struggles with writing explanations and math reasoning** | Continued use of math journaling and share time in which classmates critique each other can help strengthen this. |
| **Student struggles with creating a fractional model for multiplying fractions** | Use manipulatives, such as folding a piece of paper vertically and horizontally to find the fraction of a fraction |
| **Student misunderstands the concept of multiplication of a fraction resulting in a product less than what you started with** | Use number lines to show that a fraction falls between 0 and 1; and then make the connection for students that the product, when multiplying by a fraction, must fall between zero and the original number. |